1212-18-505-021

Code No. 8192 / Core

FACULTY OF SCIENCE

M.Sc. II – Semester Examination, May / June 2019

Subject: Mathematics / Applied Maths

Paper – I

Galois Theory

Time: 3 Hours **Max.Marks: 80**

Note: Answer all questions from Part-A and Part-B.
Each question carries 4 marks in part-A and 12 marks in Part-B.

PART – A (8x4 = 32 Marks)
[Short Answer Type]

1 Let $f(x)\in F[x]$ be a polynomial of degree >1. If $f(\alpha) = 0$ for some of $\alpha \in F$ then f(x) is reducible over F.

2 If E is a finite extension of F then show that E is an algebraic extension of F.

3 Let $F = \frac{z}{(2)}$ then prove that the splitting field of $x^3+x^2+1 \in F[x]$ is a finite field with eight elements.

4 Let F be a finite field then prove that the number of elements of F is p^n for some positive integer n and p is prime number.

5 Prove that $G = G\left(\frac{\mathbb{C}}{R}\right)$ is a group of R-automorphisms of ¢ and $|G| = 2$.

6 Prove that the group $G\left(\frac{Q(\alpha)}{Q}\right)$, where $\alpha^5 = 1$ and $\alpha \neq 1$ is isomorphic to the cyclic group of order 4.

7 Prove that $\phi_8(x)$ and x^8-1 have the same Galois group, namely $\left(\frac{Z}{(8)}\right)^{*} = \{1, 3, 5, 7\}$, the Klein four group.

8 Show that $x^5 - 9x + 3$ is not solvable by radicals over Q.

PART – B (4x12 = 48 Marks)
[Essay Answer Type]

9 a) Let $F\subseteq E\subseteq K$ be fields. If $[K:E] < \infty$ and $[E:F] < \infty$ then show that
 i) $[K:F] < \infty$
 ii) $[K:F] = [K:E]\,[E:F]$

OR

b) For any field K the following are equivalent
 i) K is algebraically closed
 ii) Every irreducible polynomial in K[x] is of degree 1
 iii) Every polynomial in K[x] of positive degree factors completely in K[x] into linear factors
 iv) Every polynomial in K[x] of positive degree has atleast one root in K.

10 a) Prove that the degree of the extensions of the splitting field of $x^3 - 2 \in Q[x]$ is 6.

OR

b) If E is a finite separable extension of a field F then prove that E is a simple extension of F.

11 a) Let H be a finite subgroup of the group of automorphisms of a field E then $[E:E_H] = |H|$.

OR

b) State and prove fundamental theorem of Galois Theory.

12 a) Let F contains a primitive n^{th} root w unity then the following are equivalent.

I) E is a finite cyclic extension of degree n over F

ii) E is the splitting field of an irreducible polynomial $x^n - b \in F[x]$

Further more $E = F(\alpha)$ where α is a root of $x^n - b$.

OR

b) Prove that the following are equivalent statements:

i) $a \in R$ is constructible form Q

ii) (a, 0) is a constructible point from QxQ

iii) (a, a) is a constructible point form QxQ

iv) (0, a) is a constructible point from QxQ.

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1212-18-505-021

Code No.8194/CORE

FACULTY OF SCIENCE

M.Sc. II-Semester Examinations, May/June 2019

Subject: Mathematics / Applied Maths

Paper-II

Lebegue Measure and Integration

Time : 3 Hours **Max. Marks : 80**

Note : Answer all questions from Part-A and Part-B. Each question carries 4 marks in Part-A and 12 marks in Part-B.

PART – A (8 x 4 = 32 Marks)

1 If E_1 and E_2 are any two measurable subsets of $\mathbb{R}$ then show that $E_1 U E_2$ is also measurable.

2 If F : E → $[-\infty, \infty]$ is a measurable function then show that |f| is a also a measurable function. (Here E is a measurable subset of $\mathbb{R}$).

3 Define simple function. Show that every simple function defined on a measurable set $E \subseteq \mathbb{R}$ can be written as the linear combination of characteristic functions of a finite collection of measurable sets.

4 Suppose f is a bounded measurable function defined on E with $m(E) < \infty$. If E_1, E_2 are any two disjoint measurable sets, then show that $\int_{E_1 \cup E_2} f = \int_{E_1} f + \int_{E_2} f$

5 If F : [a, b] → IR is of bounded variation on [a, b], then show that $P_a^b - N_a^b = f(b) - f(a)$.

6 Let f_n : [0, 1] → IR be given by

$$f_n(x) = \begin{cases} 1 & \text{if } x \in \left[\frac{k}{2^r}, \frac{k+1}{2^r}\right] \\ 0 & \text{otherwise} \end{cases}$$

Where $n = 2^r + k$, $0 \leq 0\ k < 2r$. Then show that $f_n \to f$ [measure] on [0,1] where $f(x) = 0\ \forall x \in [0,1]$.

7 Suppose f is integrable on [a, b] and $\int_{[a,x]} f = 0$ for all $x \in [a,b]$ then show that f(x) = 0 a.e on [a,b].

8 Let a, b, t are non negative real numbers and $1 \leq p < \infty$. Then show that $a^p + pt\ ba^{p-1} \leq (a + tb)^p$.

PART – B (4 x 12 = 48 Marks)

9 a) i) Suppose $\{E_n\}$ is a sequence of measurable sets in IR such that
i) $m(E_1) < \infty$ and
ii) $E_n \supseteq E_{n+1} \forall n \geq 1$ then show that $\lim_{n\to\infty} m(E_n) = m\left(\bigcap_{n=1}^{\infty} E_n\right)$

ii) Give an example to show that $m(E_1) < \infty$ is essential in the above result (i).

OR

b) Show that there exist bounded non measurable set in IR.

10 a) State and prove bounded convergence theorem.

OR

b) Suppose f and g are Lebesgue measurable functions defined on E then show that
i) $\int_E f+g = \int_E f + \int_E g$
ii) $\int_E cf = c\int_E f$ Where $C \in \mathbb{R}$

11 a) State and prove Vitali covering lemma.

OR

b) i) If $f : [a, b] \to$ IR is a monotonic function, then show that f is of bounded variation [a, b].
ii) If $f : [a, b] \to$ IR is of bounded variation on [a, b] then show that f is of bounded variation on [a, x] where a < x < b.

12 a) Let f, g $\in L^p[0, 1]$ where K p < ∞. Then show that $\|f+g\|_p \leq \|f\|_p + \|g\|_p$

OR

b) Suppose f is Lebesgue integrable on [a, b]] and F is its indefinite integral. Then show that F is continuous on [a, b] and also F is of bounded variation on [a, b].

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1212-18-505-021

Code No. 8197 / CORE

FACULTY OF SCIENCE

M.Sc. II – Semester Examination, May / June 2019

Subject: Mathematics

Paper – III
Complex Analysis

Time: 3 Hours **Max.Marks: 80**

Note: Answer all questions from Part-A and Part-B.
Each question carries 4 marks in part-A and 12 marks in Part-B.

PART – A (8x4 = 32 Marks)
[Short Answer Type]

1 Show that analytic function cannot have a non-zero constant absolute value without reducing to a constant.

2 Find all values of z such that (1) $e^z = 1+\sqrt{3i}$ (2) $e^z = 1+i$.

3 Let C be the triangle with vertices at 0, 3i, -4. Prove that $\left|\int_C (e^z - \bar{z})\,dz\right| \le 60$.

4 Let C be the circle $z = e^{i\theta}$ $(-\pi \le \theta \le \pi)$. For $a \in \mathbb{R}$, prove that $\int_C \frac{e^{az}}{z}\,dz = 2\pi i$. And hence compute $\int_0^{\pi} e^{a\cos\theta} \cos(a\sin\theta)\,d\theta$.

5 Let C: $w = e^{i\varphi}$ $(-\pi \le \varphi \le \pi)$ and $e^{\frac{z}{2}(w-\frac{1}{w})} = \sum_{n=-\infty}^{\infty} J_n(z) w^n$ $(0 < |w| < \infty)$. Prove that $J_n(z) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-i(n\varphi - z\sin\varphi)}\,d\varphi$.

6 Let C denote the circle $|z| = 1$, taken counter clock wise, show that $\int_C \exp\left(z+\frac{1}{z}\right)dz = 2\pi i \sum_{n=0}^{\infty} \frac{1}{n!(n+1)!}$.

7 State and prove Jordan Lemma.

8 Evaluate $\int_0^{2\pi} \cos^{2n}\theta\, d\theta$ (n = 1, 2, ...).

PART – B (4x12 = 48 Marks)
[Essay Answer Type]

9 a) State and prove the necessary condition for differentiability of a complex valued function f(z).

OR

..2

b) Find the most general harmonic polynomial of the form $u = ax^3+bx^2y+cxy^2+dy^3$ and hence find the corresponding harmonic conjugate. Express f in terms of z.

10 a) State and prove Gauchy – Goursat theorem.

OR

b) i) State and prove Cauchy's integral formula.

ii) State and prove Morera's theorem.

11 a) State and prove Laurent theorem.

OR

b) Let C_N denote the positively oriented boundary of the square whose edges lie along the lines

$$x = \pm\left(N+\frac{1}{2}\right)\pi \text{ and } y = \pm\left(N+\frac{1}{2}\right)\pi$$

where N is a positive integer, show that $\int_{C_N} \frac{dz}{z^2 \sin z} = 2\pi i \left(\frac{1}{6}+2\sum_{n=1}^{N}\frac{(-1)^n}{n^2\pi^2}\right)$.

12 a) Evaluate $\int_{-\infty}^{\infty} \frac{1}{x^4-1}\,dx$.

OR

b) State and prove argument principle.

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1212-18-505-021

Code No.8202/CORE

FACULTY OF SCIENCE

M.Sc. II-Semester Examinations, May/June 2019

Subject : Mathematics

Paper-IV : Topology

Time : 3 Hours **Max. Marks : 80**

Note : Answer all questions from Part-A and Part-B. Each question carries 4 marks in Part-A and 12 marks in Part-B.

PART – A (8 x 4 = 32 Marks)

1 Give an example to show that the union of two topologies on a set X need not be a topology.
2 Define interior of a set A in topological space (X,T). Also show that the interior of A is the largest open subset of A.
3 Show that any continuous image of a compact space is compact.
4 Show that every sequentially compact metric space is totally bounded.
5 Show that a topological space is a T_1 – space if each one point set is a closed set.
6 Define a complete regular space.
7 Show that a topological space X is connected if ϕ and X are the only subsets of X which are both open and closed.
8 Let X be a topological space and A is subspace of X and is connected. If B is any subspace of X such that $A \subset B \subset \overline{A}$ then show that B is also connected.

PART – B (4 x 12 = 48 Marks)

9 a) Let (X, T) be a topological space and A is a subset of X then
i) Define boundary of A
ii) Show that A is the disjoint union of the set of all boundary points of A and the set of all interior point of A.

OR

b) State and prove Lindelof's theorem.

10 a) Show that a topological space is compact if and only if every class of closed sets with finite intersection property has nonempty intersection.

OR

b) Show that a complete totally bounded metric space is compact.

11 a) Show that a one to one continuous mapping of a compact space onto a Housdorff space is a homeomorphism.

OR

b) State and prove Tietze extension theorem.

12 a) Show that a subspace of the real line IR is connected if and only if it is an interval.

OR

b) i) Define box topology.
ii) If β, ζ are the open bases for topological spaces X and Y respectively then show that $D = \{B \times C \mid B \in \beta, C \in \zeta\}$ is a base for the box topology on X x Y.

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1212-18-505-021

Code No. 8203 / CORE

FACULTY OF SCIENCE

M.Sc. II – Semester Examination, May / June 2019

Subject: Mathematics / Applied Mathematics

Paper – V
Theory of Ordinary Differential Equations

Time: 3 Hours **Max.Marks: 80**

Note: Answer all questions from Part-A and Part-B.
Each question carries 4 marks in part-A and 12 marks in Part-B.

PART – A (8x4 = 32 Marks)
[Short Answer Type]

1 Prove that the functions $x_1(t) = t^2$ and $x_2(t) = t\,|t|$ are linearly independent on $-\infty < t < \infty$ but they are linearly dependent on the intervals $-\infty < t \le 0$ and $0 \le t < \infty$.

2 Let $\varphi_1, \varphi_2, \ldots, \varphi_n$ be n linearly independent solutions of the homogeneous equation $L(x) = x^{(n)} + b_1(t)\, x^{(n-1)} + \ldots + b_n(t)\, x = 0,\ t \in I$ exists on I. Let x_p denote any particular solution of the non-homogeneous equation $L(x) = h(t)$ existing on I then prove that any solution of x of $L(x) = h(t)$ is given by $x(t) = x_p(t) + c_1 \varphi_1(t) + \ldots + c_n \varphi_n(t),\ t \in I$ where $c_1, c_2, \ldots, c_n$ are n arbitrary constants (real or complex).

3 Determine the constants L, K and h for the IVP $x' = x^2$, $x(0)=1$, $R=\{t,x) / |t| \le 2.\ |x-1| \le 2\}$.

4 Assume that f(t) and g(t) are non-negative continuous functions for $t \ge t_o$. Let $k > 0$ be a constant then prove that the inequality $f(t) \le k + \int_{t_o}^{t} g(s) f(s)\, ds$, $t \ge t_o$ implies the inequality

$f(t) \le k \exp\left(\int_{t_o}^{t} g(s)\, ds\right), t \ge t_o$.

5 Define:
i) Upper solution
ii) Lower solution of the IVP $x' = f(t, x)$, $x(t_o) = x_o$ on $[t_o, t_o + h]$.

6 State and prove Bihari's inequality.

7 Show that every second order homogeneous linear differential equation can be reduces to self adjoint form.

8 $y_1(x) = x$ is one solution of the second order differential equation $x^3 y'' - x y' + y = 0$, $x > 0$ then find second solution $y_2(x)$.

PART – B (4x12 = 48 Marks)
[Essay Answer Type]

9 a) Let $\varphi_1, \varphi_2, \ldots, \varphi_n$ be n linearly independent solutions of the homogeneous equation $L(x) = (x)^n + b_1(t)\, x^{(n-1)} + \ldots + b_n(t)\, x = 0,\ t \in I$ existing on I. Let the real or complex valued function h be defined and continuous on I. Further, assume that $w(t) = w(\varphi_1, \varphi_2, \ldots, \varphi_n)$ and $w_k(t)$ denotes the determinant w(t) with k^{th} column replaced by n elements 0, 0, …,1. The show that a particular solution $x_p(t)$ of $L(x) = h(t)$ is given by $x_p(t) = \sum_{k=1}^{n} \varphi_k(t) \int_{t_o}^{t} \frac{w_k(s)\, h(s)}{w(s)}\, ds,\ t \in I.$

OR

b) Let $b_1, b_2, \ldots, b_n$ be real or complex valued functions defined and continuous on an interval I and $\varphi_1, \varphi_2, \ldots, \varphi_n$ are n functions of the equation $L(x) = x^{(n)} + b_1(t)\, x^{(n-1)} + \ldots + b_n(t)\, x = 0,\ t \in I$ existing on I. Then prove that these n solutions are linearly independent on I if and only if $w(t) \neq 0$ for every $t \in I$.

10 a) Let f(t,x) be continuous and be bounded by L and satisfy Lipschitz condition with Lipschitz constant K on the closed rectangle R then prove that the successive approximations x_n, n = 1, 2, … given by $x_n(t) = x_o + \int_{t_o}^{t} f((s, x_{n-1}(s))\, ds$, n = 1, 2, … converge uniformly on an interval $I = |t - t_o| \leq h$, $h = \min\{a, \frac{b}{L}\}$, to a solution x of the IVP $x' = f(t,x)$, $x(t_o) = x_o$. Also, prove that this solution is unique.

OR

b) Prove that the IVP $x' = f(t,x)$, $x(t_o) = x_o$ has a unique solution defined on $t_o \leq t \leq t_o + h$, h > 0, if the function f(t, x) is continuous in the strip $t_o \leq t \leq t_o \leq h$, $|x| < \infty$ and satisfies the Lipschitz condition $|f(t,x_1) - f(t, x_2)| \leq K\, |x_1 - x_2|$, k > 0, K being Lipschitz constant.

11 a) Let $m \in C\{[t_o, t_o+h), R\}$, $f \in C\{[t_o, t_o+h) \times R, R\}$ and $D^+m(t) = \lim_{h \leftarrow 0} \sup \frac{1}{h}[m(t+h) - m(t)] \leq f(t, m(t))$, $t \in (t_o, t_o+h)$ then prove that $m(t_o) \leq x_o$ implies $m(t) \leq r(t)$, $t \in [t_o, t_o+h)$ where r(t) is the maximal solution of $x' = f(t, x)$, $x(t_o) = x_o$ existing on $[t_o, t_o+h]$.

OR

b) Let $f \in C\ [I \times R, R]$, v_o, w_o be lower and upper solutions of $x' = f(t, x)$, $x(t_o) = x_o$ such that $v_o \leq w_o$ on $I = [t_o, t_o+h]$. Suppose, further that $f(t,x) - f(t,y) \geq -M(x-y)$ for $v_o \leq y \leq x \leq w_o$ and $M \geq 0$. Then prove that there exists monotone sequence $\{v_n\}$, $\{w_n\}$ such that $v_n \to v$ and $w_n \to w$ as $n \to \infty$ uniformly and monotonically on I and that v, w are minimal and maximal solutions of $x' = f(t,x)$, $x(t_o) = x_o$ respectively.

12 a) State and prove Sturm separation theorem.

OR

b) State and prove Sturm Picone Theorem.

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